Complementarity in vectorial quantum light interference

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1. Introduction

The principle of complementarity, stating that quantum systems share mutually exclusive properties, is a central concept in physics. The arguably most recognized appearance of such complementarity is wave–particle duality, which limits the coexistence of wave and particle properties of a quantum object. In two-way interferometry, such as the celebrated double-slit experiment, the dual wave–particle quantum character can be quantified as [1]

$$D_0^2 + V_0^2 \leq 1. \tag{1}$$

Here D_0 is the path predictability, representing the particle aspect or "which-path information" (WPI) of the object, and V_0 is the interferometric intensity fringe visibility, describing the wave aspect of the object. Photons, however, may exhibit interference not only in terms of intensity fringes but also (or merely) in terms of polarization-state fringes [2], a distinctive quality of vectorial light fields that has no correspondence in scalar-light interferometry. The manifestation and quantification of quantum complementarity under such interferometric polarization-state modulation is therefore of fundamental interest. In this work, we explore polarization modulation in double-slit interference and establish three general complementarity relations for quantized vector-light fields, revealing new foundational features of the dual wave–particle nature of photons [3,4].

2. Results and discussion

It can be shown that any vectorial quantum light in two-slit interference obeys the complementarity relations [3]

$$D_0^2 + V_S^2 \leq 1, \quad D_S^2 + V_0^2 \leq 1. \tag{2}$$

Here the Stokes visibility V_S is the vector-light generalization of the usual intensity fringe visibility V_0 encountered in the scalar-light context, that also characterizes the visibility of polarization-state fringes in the detection plane. The Stokes distinguishability D_S in turn describes the polarization-state difference of the light field at the two slits, thus providing another central measure to quantify the WPI in the system in addition to the path predictability D_0. At the single-photon level, the complementarity relations in Eq. (2) reflect two very different, fundamental aspects of wave–particle duality of the photon [3] having no correspondence within the scalar-light framework of Eq. (1). Especially, in the vector-light scenario the path predictability D_0 does not couple to the intensity visibility V_0 but instead to the Stokes visibility V_S which accounts also for the polarization-state modulation. This feature highlights an essential quality concerning wave–particle duality of vectorial quantum light fields.

Let us consider further a quantum plane-wave field that contains two orthogonal polarization modes and whose degree of polarization is P [2]. The two modes are separated and directed towards the slits where arbitrary local unitary operations can be performed, which allows one to control the intensity and/or polarization modulation in the detection plane where the modes are eventually superposed. In this case we discover that [4]

$$P^2 = D_0^2 + V_S^2, \tag{3}$$

forming a fundamental link among the path predictability at the slits, the Stokes visibility in the observation plane, and the degree of polarization of the initial (undivided) light field. It implies that a change of D_0 or V_S alters its complementary partner so that the sum of their squares strictly equals P^2. We can thereby interpret P as specifying the "complementarity strength" between D_0 and V_S. Equation (3) also indicates that $D_0, V_S \leq P$, dictating that the degree of polarization sets the upper bound both for the path predictability and for the Stokes visibility.

Hence our work, together with its current extension to more complex topologies involving geometric phases, quantum polarization uncertainties, as well as materials displaying nonlinear interactions, provides deeper insights into foundational quantum photonics.

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4. References

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